Abstract—In this study, a two phase heuristic bubble approach is presented for solution of the Location-Routing Problems (LRP), which has two subsections called Location Allocation Problem (LAP) and Vehicle Routing Problem (VRP). The proposed two phase heuristic bubble approach is designed to solve these two problems separately. A new heuristic approach is Heuristic Bubble Polygon (HBP) is proposed for LAP and Heuristic Bubble Algorithm (HBA) was proposed is used as a solution to VRP. The benefit of the proposed two phase heuristic bubble approach are studied via simulations and the obtained results are compared with Jokar method and Sahraeian methods.

I. INTRODUCTION

The large part of expenses of companies are represented by logistic costs. Depot location and vehicle routing which are crucial selections to reduce the logistic costs are decided separately however this decision method causes suboptimal solutions [1]. Thus, one of the most difficult problems in supply chain management consists in an appropriate coordination between depot location and vehicle routing [2]. This problem is divided in itself two problems are location allocation problems (LAP) and vehicle routing problems (VRP). The location allocation problems are selecting the best locations for depots from a given group of candidate sites to optimize the cost of assigning customers to the depots and the total depot opening cost. The vehicle routing problems are to design the optimal routes from a given depot to geographically scatter customers [3]. The solution of this type of problem is in two phases are the decision of depot location for a long term and the routing decisions to satisfy customer requests.

The Location-Routing Problems (LRP) can be described as the vehicle routing problems in which the locations and optimal numbers of the depots are determined simultaneously with the distribution routes and the vehicle schedules to optimize the total system costs [4]. Facility location and vehicle routing aspects are addressed simultaneously in the LRP. In order to formulate and solve some distribution network design aspects, many LRP s such as multi depot LRP and capacitated LRP have been introduced in such a manner take into account to minimize depot fixed and operating costs routing costs, vehicle fixed cost to satisfy the customer requests and some given depot and vehicle capacity constraints [5]. The considered objectives are inadequate for reducing economic returns in more traditional transportation optimization models. Therefore, multi-objective decision approach is used involves three objectives. The first objective is that optimization of the fixed costs for depots and vehicles, which is a medium term economic cost function. The second objective is that optimization of the estimates for the operative cost which is short term economic cost function. The last objective is that maximization of covered request which is a non-monetary profit function. By the last objective function, the transportation cost that is optimized demonstrably concerns transport from the facilities to depots is quantified while the optimal routing from depots to consumers is included in the maximization of covered request [6].

The LRP has many applications for a wide fields or diversity such as food and drink distribution, waste collection, parcel delivery, newspapers delivery, military applications and several consumer goods distribution [7]. Additionally, one application of LRP in the real life applications is the combined problem of bus stop selection and bus route generation [8].

The LRP is an NP-hard problem and the instances have numerous customers cannot be solved to optimality within a suitable time. Therefore, in order to find a close optimal solution in suitable computational time, many heuristic, metaheuristic and stochastic algorithms have been introduced [9]. In the literature, an analytical survey of the LRP is presented in [10] and classical heuristic algorithms for the solution of LRP are presented in [11] and [12]. In [13, 14], various metaheuristic algorithms based on Tabu Search have been proposed for the solution of LRP. Simulated annealing based on greedy randomized adaptive search procedure (GRASP) is used in [15] to solve the LRP. Moreover, Genetic algorithms in [16] and variable neighborhood search algorithms in [17] are presented for the LRP.

In this study, a two phase heuristic bubble approach is presented for solution of the LRP. The LRP has two subsections which are Location Allocation Problem and Vehicle Routing Problem. The two phase heuristic bubble approach is designed to solve these two problems separately. A new heuristic approach is Heuristic Bubble Polygon (HBP) is proposed for LAP and Heuristic Bubble Algorithm (HBA) was proposed is used to VRP. For case studies, 2 standard instances (CLRIP) are studied to express
benefit of the two phase heuristic bubble approach and then the results of two phase Heuristic bubble approach are compared with Jokar method and Sahraeian method.

The outline of the paper is organized as follows: Section II defines the location-routing problem and gives a generic LRP formulation. Section III presents a new heuristic approach is Heuristic Bubble Polygon (HBP), Section IV represents the proposed Heuristic Bubble Algorithm (HBA), Section IV demonstrates case studies, and Section V involves conclusions and remarks.

II. MULTI DEPOT LOCATION-Routing PROBLEM

The multi-depot location-routing problem is defined in [18]. In this problem, only the single production LRP is interested. It is assumed that location, request, and number of customers, the number and location of all potential depots, as well as the type and capacity of vehicles are given in a logistic system. The distribution and routing must be planned to satisfying for various constraints which are the request of each customer can be satisfied, each customer is served by exactly one vehicle, the total request on each route is not more than the capacity of the vehicle appointed to that route, and each route starts and finishes at the same depot.

The model aims to optimize the total cost that is utilized in the multi-depot location-routing problem and can be formulated as follows.

\[
\min \left( \sum_{i \in S} \sum_{j \in S} \sum_{k \in V} C_{ij} \times X_{ijk} + \sum_{k \in V} (G_k \sum_{j \in H} \sum_{r \in G} X_{rjk}) \right) + \sum_{r \in G} F_r \times Z_r 
\]

where \( G \) is the set of \( R \) possible sites of candidate facilities, \( H \) is the set of \( N \) customers to be served, \( S \) (\( \{G\} \cup \{H\} \)) is the set of all possible sites and customers, \( V \) is the set of \( K \) vehicles proper for routing from the depots, \( C_{ij} \) is the mean annual cost of traveling from node \( i \)th to node \( j \)th, \( C_k \) is the annual cost of acquisition vehicle \( k \), \( F_r \) annual cost of establishing and management a facility at site \( r \), and if vehicle \( k \) travels from \( i \)th to \( j \)th distribution centers \( X_{ijk} \) is equal to 1, or it is equal to 0 and if a facility is established at site \( r \) is equal to 1, or it is equal to 0. The multi-depot LRP can be resolved according to (1) and can be subject to following equations.

\[
\sum_{k \in V} \sum_{i \in S} X_{ijk} = 1, \quad \forall j \in H \tag{2}
\]

\[
\sum_{j \in H} \sum_{i \in S} q_j \times X_{ijk} \leq Q_k, \quad \forall k \in V \tag{3}
\]

\[
\sum_{i \in S} X_{ipk} - \sum_{j \in S} X_{pjk} = 0, \quad \forall k \in V, p \in S \tag{4}
\]

\[
\sum_{r \in G} X_{rjk} \leq 1, \quad \forall k \in V \tag{5}
\]

\[
\sum_{k \in V} X_{rmk} + Z_r + Z_m \leq 2, \quad \forall m = 1, ... R, \quad r \in G \tag{6}
\]

\[
\sum_{k \in V} X_{rjk} - Z_r \geq 0, \quad \forall r \in G \tag{7}
\]

\[
\sum_{j \in H} X_{rjk} - Z_r \leq 0, \quad \forall k \in V, r \in G \tag{8}
\]

\[
R_i + R_j + (R + N) \sum_{k \in V} X_{ijk} \leq R + N - 1, \forall r, j \tag{9}
\]

\[
X_{rjk} = 0 \text{ or } 1, \quad \forall i, j \in S, \quad k \in V \tag{10}
\]

\[
Z_r = 0 \text{ or } 1, \quad \forall r \in G \tag{11}
\]

where \( q_j \) is the mean number of units demanded by customer \( j \), \( Q_k \) is the loading capacity limit of each vehicle, \( d_{ij} \) is the distance from node \( i \)th to node \( j \)th and \( R_i \) are continuous variables used in the sub tour breaking constraints.

Equation (1) is the objective function that is the basis of the multi depot LRP and its target is optimizing the total cost of location and operating depots and routing and acquiring of the vehicles. Equation (2) expresses that each customer can be visited once by exactly one vehicle. Equation (3) guarantees that each vehicle cannot exceed the capacity of the vehicle and (4) is route continuity constraints that ensures every address is assigned to exactly one vehicle and is left by the identical vehicle. Equation (5) guarantees that each vehicle is routed from only one depot. Equations (6) expresses that any two depots have not connection between each other. Equation (7) and (8) claims that a vehicle is routed from a depot if and only if that depot is opened. Equation (9) is called as the sub tour elimination constraints which provides that each tour must include a depot from which it is stating point of route. The last two equations are the binary necessity on the decision variables.

III. HEURISTIC BUBBLE POLYGON

A new heuristic approach called Heuristic Bubble Polygon (HBP) is proposed here to determine the location of depots for supply optimal routes. The bubbles are served from customers are routed according to optimal routes and the bubbles are converged to the center of mass of routes weighted the size of bubbles, iteratively. Thus, the bubbles are collapsed into the center of plane or map. The places where collapsed bubbles are assigned to depots at the end of the all iteration.

Initially, the senders, which one unit away from customers for the Euclidian space or one km away from customers for the real life location routing problems, are generated. Consequently, a sender is generated for each customer. The distance from one sender to all customers and other senders for each sender is calculated the distances are saved in the distance matrix. In the distance matrix, the distance from the sender to its customer is determined one unit, the distance from the sender to another customers is determined one unit away from the distance from its customer to another customers and the
distance from the sender to another senders is determined two unit away from the distance from its customer to the customer of another senders. The unit of distance is kilometer for the real life location routing problems.

Then, the customers are routed with their senders. The routing is performed that one random point is selected randomly in the Euclidean space or map and the circle is drawn around the selected point. The two parameters which are mass cluster distance determines the radius of circle and mass point size determines the maximum number of customers in the drawn circle. The customers in the drawn circle are merged by using the merge operator of Heuristic Bubble Algorithm which will be explained in the next section. After all customers are merged, the merged customers with their senders become routes for the each vehicle. In the route, the vehicle picks up bubble, is called as order, from one sender and delivers the bubble to its customers, then delivers the bubble to next customer from the sender of next customer. All bubbles are served to the customers by the vehicle in this way. The objective functions of all routes are calculated. The objective function that is used in the HBP is given as

$$\min \left( \frac{\sum_{r=1}^{R} (Q - V_r) \times D_r}{T} \right)$$  \hspace{1cm} (12)$$

where $Q$ is the maximum capacity of vehicle, $V_r$ is the number of bubble that is served $r^{th}$ road, $D_r$ is the distance of $r^{th}$ road is taken from distance matrix, $R$ is the length of the route and $T$ is the amount of served bubbles in the route. The route having optimum objective function is selected and saved. After, the point is selected randomly and the best route is determined around this point. This process continues in an iterative manner until all customers are routed with their senders.

After all customers are routed, the center of mass of routes are calculated according to the relation between customers and bubbles. The center of mass can be found as

$$x^{(k)}_{\text{COM}} = \frac{\sum_{i=1}^{N} (x_i, V_i)}{V}$$  \hspace{1cm} (13)$$

where $x_i$ is the $i^{th}$ customer in the route and $V_i$ is the number of bubble that is served to the $i^{th}$ customer, $V$ total number of bubble and $N$ is the number of customer in the route. The center of mass of the routes becomes the new customers in the HBP and the served bubbles to the new customers are equal to the total number of bubble in the route. The new senders are determined, the distance matrix of new senders is calculated, the routes are generated and the center of mass of new routes are calculated for the new customers. The new center of mass of routes becomes the new customers of the next step which means that all processes are to be carried out repeatedly until the stopping criteria of HBP is satisfied. Therefore, the number of customers and their senders’ decreases and the HBP is approximated to the center plane. The HBP until the number of last remaining customers and senders is reached to the parameter is called as stop orders which is the stopping criteria. Finally, the determined senders are assigned to the depots.

The flowchart of the Heuristic Bubble Polygon is shown step by step in Fig.1.

![Flowchart of Heuristic Bubble Polygon](image)

IV. HEURISTIC BUBBLE ALGORITHM

Heuristic Bubble Algorithm (HBA), which is a novel heuristic algorithm, was generated with motivated from division and union of bubbles and inspiration from nature to get optimum solution daily macro routing problems. In the HBA, the bubbles which are called as the demands are served to customers are toured from the depots to customers by vehicles. The HBA is used to find optimal route has minimum objective function for each vehicle iteratively. The best bubble group is found until all bubble groups are served to customers in all iteration. In previous studies, the HBA was used to find solution in [19, 20] for VRRPD and in [21] for PDPTW. In this study, the HBA is used for the routing decisions to satisfy customer request in LRP with improvements in the five operators are merge, join, result elimination, join and swap operators of HBA.

1) Merge Operator

The merge operator which is the first phase of each iteration finds all combinations of customers. The maximum and minimum numbers of combination is entered to the algorithm parametrically. The combinations of customers becomes routes for each vehicle. Then, the depot of each routes are determined by that all candidate depots are set to routes and the objective functions of each route with determined depots are calculated. The route has optimum objective function value for each route is determined and the depot of each route is selected. The
bubbles from depot to customers are loaded to the vehicle according to route of vehicles. The capacity of vehicles must not be exceeded while the bubbles are loaded to the vehicle at the determined depots. Moreover, the stock of depot is controlled by that if the stock of depot is finished, the depot of route can be changed. After all customers are merged and the depot of each route is set, the objective function of each route is calculated and the route has optimum objective function is selected and the selected route is utilized in join operator. After the join operator is performed, the customers are merged and the depot of merged customers are determined again. The remained bubbles are loaded the vehicle at the determined depots. The capacity constraint and the situation of stock of depot is controlled then the route has optimum objective function is selected and the selected route is utilized in the join operator. The merge operator performs iteratively until all bubbles are served to the customers.

2) Join Operator
The join operator which is the second phase of algorithm aims to optimize the objective function by adding extra customers and loading bubbles of extra customers from depot to merged route. The maximum and minimum numbers of extra customers can be added to the merged route parametrically as same as the merge operator. The join operator performs in the following way: The extra customers which are not merged in the merge operator is added to the between the customers and end of merged route, all adding situations are saved and the objective function is calculated for each situation. The situation has optimum objective function is determined and the extra customer is added for selected route according to this situation. At this stage, two parameters are utilized to decide which customers can be added to the selection of extra forts in the join operator. Firstly, the $\alpha$ distance parameter determines the maximum distance of extra customer to the customer of merged route. Secondly the $\beta$ distance parameter determines the maximum length of route when the extra customers are added. The extra customers are not added to the merged route if one of two parameters states are not satisfied. Therefore, the number of possible extra forts is diminished to addition in the join operator and the algorithm is accelerated.

After the extra customer is added to the merged route, the bubble of extra fort is loaded to vehicle of route. The capacity constraint and the stock situation of depots is controlled to prevent capacity exceed in the vehicles and stock out in the depot. If there is no problem in the new routes, the objective function of route with new customer is calculated. If the objective function is optimized, the extra customer and its bubbles remain to be added for the merged route. The join operator performs until there is no optimization in the objective function.

3) Result Elimination Operator
The result elimination operator is aimed that optimize the number of route and the total distance of routes then all customers are routed and the all bubbles of customers are served from depots in the merge and join operators. The routes are compared according to objective function and the route has maximum objective function is determined and exterminated in this operator. The customers are their bubbles which are in the exterminated route are added to the overall routes as same as join operator. At this stage, the capacity constraint and the stock situation of depots is controlled for the new routes. After new routes are verified, the objective function of route with new customers and their bubbles is calculated. If the objective function of routes is optimized, the alteration in the new route is accepted. Another bubbles in the exterminated route are served by using other routes with optimization of objective function. The result elimination operator performs iteratively which means that the route having the maximum function is determined again and the customer and their bubbles of the worst routes are served by other routes for optimize objective function. If there is no optimization in the objective function, this operator is terminated. In the algorithm, the result elimination operators is performed twice which are after merge and join operators and after the swap operator.

4) Split Operator
The split operator utilizes the division and union of bubbles thus the total distance of routes is reduced in this operator. The split operator targets that the bubbles are delivered from one depot to two different customers in the two routes to be delivered by a single route. The route having the worst objective function is destroyed as same as the result elimination operator in this operator. The bubbles in destroyed route are loaded to another vehicle depart from the depot where the bubbles must be served here by the capacity constraint is considered. Thus, the customer is expelled from the route and the route having the worst objective function is reduced. The objective function is calculated and if the objective function is optimized, the alteration in the routes is saved. The split operator performs until there is no improvement in the objective function.

5) Swap Operator
The final operator of HBA is swap operator provides randomness to the algorithm and aims to optimize the total distance of routes. The two different customers in the different two routes are selected randomly. These customers and their bubbles are exchanged by that the capacity constraint is taken into account iteratively. The number of iteration in the swap operator is parametrical and this parameter as called as the number of swap is defined to the algorithm. At the end of each iteration, the objective function is calculated. If the objection function is optimized, the change in the routes is accepted otherwise the change is not accepted and the routes are remained in the same manner. The swap operator terminates when the number of swap is equal to the last value of swap number.

The pseudo-code for the proposed Heuristic Bubble Algorithm (HBA) in given Table I.

V. EXPERIMENTAL RESULTS AND DISCUSSIONS
In this section, the proposed two phase heuristic bubble approach is compared with the results of Jokar method and Sahreian method [22, 23] via simulations.
TABLE I. PSEUDO-CODE FOR PROPOSED HBA

<table>
<thead>
<tr>
<th>Step</th>
<th>(Operator)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(Merge Operator)</td>
<td>Determine various merged customer and choose the best merged customers with bubbles that optimize the objective function</td>
</tr>
<tr>
<td>2</td>
<td>(Join Operator)</td>
<td>Perform join operator in order until there is no improvement in the objective function (load the vehicle to the utmost)</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>Save the final bubble group (vehicle) and expressed bubbles (demands) used in instant solution (update demand list)</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>If all bubbles are served go to Step 5</td>
</tr>
<tr>
<td>5</td>
<td>(Result Elimination Operator)</td>
<td>Destroy the route having the worst objective function and join the bubbles other routes. If there is no improvement in the objective function go to Step 6</td>
</tr>
<tr>
<td>6</td>
<td>(Split Operator)</td>
<td>Divide the route having the worst objective function and serve the bubbles another route. If there is no improvement in the objective function go to Step 7</td>
</tr>
<tr>
<td>7</td>
<td>(Swap Operator)</td>
<td>Exchange the customers between the routes. Provide improvement in the objective function. If the number of swap ends go to Step 8</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>Again perform the result elimination operator. If there is no optimization in the objective function, terminate algorithm and return results</td>
</tr>
</tbody>
</table>

For comparison, 2 standard instances (CLRP) are analyzed. The two phase heuristic bubble approach is performed using the mass cluster distance as 400, the mass point size as 10, the stop orders as 3, α distance parameter as 400, β distance parameter as 100, and the number of swap as 100000. The selection of depots in candidate depots are made by that the nearest depots in the candidate depots were found by Heuristic Bubble Polygon according to the distance formula.

The results of computational experiments are tabulated in Table II, where the first column shows the name of CLRP instances with the numbers of customer and candidate depots and the second column shows the capacity of vehicle. The results of Jokar method and Sahraeian method are presented in the third and fourth columns, respectively. Moreover, the result of proposed method is represented in the fifth column.

As it is from the experiments, the proposed method has the best optimal solution for five instances within all eight instances compared to the other methods. That shows the improvement of the existing solutions of the CLRP instances in literature.

In this study, the two phase heuristic bubble approach is presented as the solution of the Location-Routing Problems (LRP). The LRP has many applications for a wide fields or diversity such as food and drink distribution, waste collection, parcel delivery, newspapers delivery, military applications and several consumer goods distribution, the combined problem of bus stop selection and bus route generation. In general, LRP has two subsections named as Location Allocation Problem (LAP) and Vehicle Routing Problem (VRP).

The proposed two phase heuristic bubble approach is designed to solve LAP and VRP separately. For this purpose, a new heuristic approach called Heuristic Bubble Polygon (HBP) is proposed as the solution of LAP, moreover, the Heuristic Bubble Algorithm (HBA) was already proposed is used as a solution to VRP. After the discussion of these two phases experiments are performed to show the benefit of the proposed approach. The results of the experimental study clearly shows that the proposed two phase heuristic bubble approach give much better results than the ones proposed when the Jokar method and the Sahraeian method are used.

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REFERENCES


VI. CONCLUSION

In this study, the two phase heuristic bubble approach is presented as the solution of the Location-Routing Problems (LRP). The LRP has many applications for a wide fields or diversity such as food and drink distribution, waste collection, parcel delivery, newspapers delivery, military applications and several consumer goods distribution, the combined problem of bus stop selection and bus route generation. In general, LRP has two subsections named as Location Allocation Problem (LAP) and Vehicle Routing Problem (VRP).

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As the future work, the application of the proposed approach will be accomplished for a real problem in Turkey, and the results will be compared with more methods from literature. Moreover, it is planned to expand the proposed approach to a software for the market.

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REFERENCES


TABLE III. THE RESULTS OF COMPUTATIONAL EXPERIMENTS

<table>
<thead>
<tr>
<th>CLRP Instance</th>
<th>Vehicle Capacity</th>
<th>Sahraeian Method</th>
<th>Jokar Method</th>
<th>Proposed Method</th>
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<tr>
<td>Christofides69-50x5</td>
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